XXIII. *Obſervations made at* Dinapoor, June 4, 1769, *on the Planet* Venus, *when paſſing over the* Sun's *Diſk*, June 4, 1769, *with Three different Quadrants, and a Two Foot reflecting Teleſcope: Communicated to the Royal Society, by the Court of Directors of the* Eaſt India *Company*.

At ſun-riſe cloudy

h ′ ″		° ′ ″
At 5 20 32 A.M.	The ſun diſingaged from the clouds when his true altitude was	1 12 11
	When Venus appeared on the ſun's diſk	
At 7 5 22	The beginning of the emerſion when the ſun's altitude was	23 35 57
At 7 23 36	The end of the emerſion when the ſun's altitude was	27 29 20
	The ſun's meridian altitude this day, was	87 56
	The latitude of the place where the obſervation was made, is 25° 27′ N°.	

The above obſervation was made by Luis Degloſs, Captain of engineers, with the aſſiſtance of J. Lang and A. Stoker.

N. B. The ſun's altitude with the hour is exactly corrected, and all the allowances made.

XXIV. *Directions*

XXIV. *Directions for making a Machine for finding the Roots of Equations universally, with the Manner of using it: By the Rev. Mr.* Rowning, *to* John Bevis, *M. D. F. R. S.*

S I R,

Read May 3, 1770.

PERUSING a discourse in the memoirs of the Royal Academy at Petersburgh, Tome vii. page 211, by the learned John Andrew de Segner, containing an universal method of discovering the roots of equations, which you was so kind as to recommend to my consideration; I found, that the author's method, as you observed to me, consisted in finding several ordinates of a parabolic curve, such, that its abscissas being taken equal to any assumed values of the unknown quantity in the equation, the ordinates corresponding to those abscissas, should be equal to the values of all the terms in the equation (when brought to one side) that is, in other words, in finding several ordinates of a parabolic curve defined by the equation proposed: in which case, as is well known, if a curve be drawn through the extremities of the said ordinates, the points upon the axis, where the curve shall cut it, will necessarily give the several values of the *real* roots

Fig. 1.

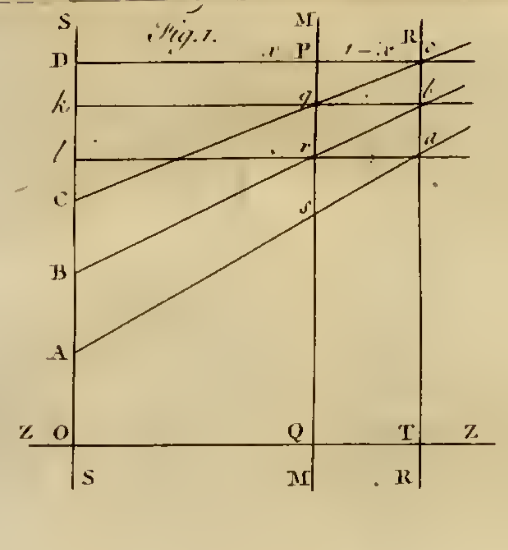


Fig. 2.

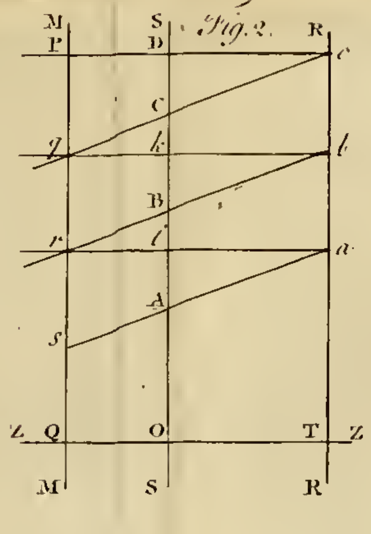


Fig. 3.

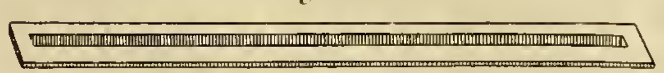
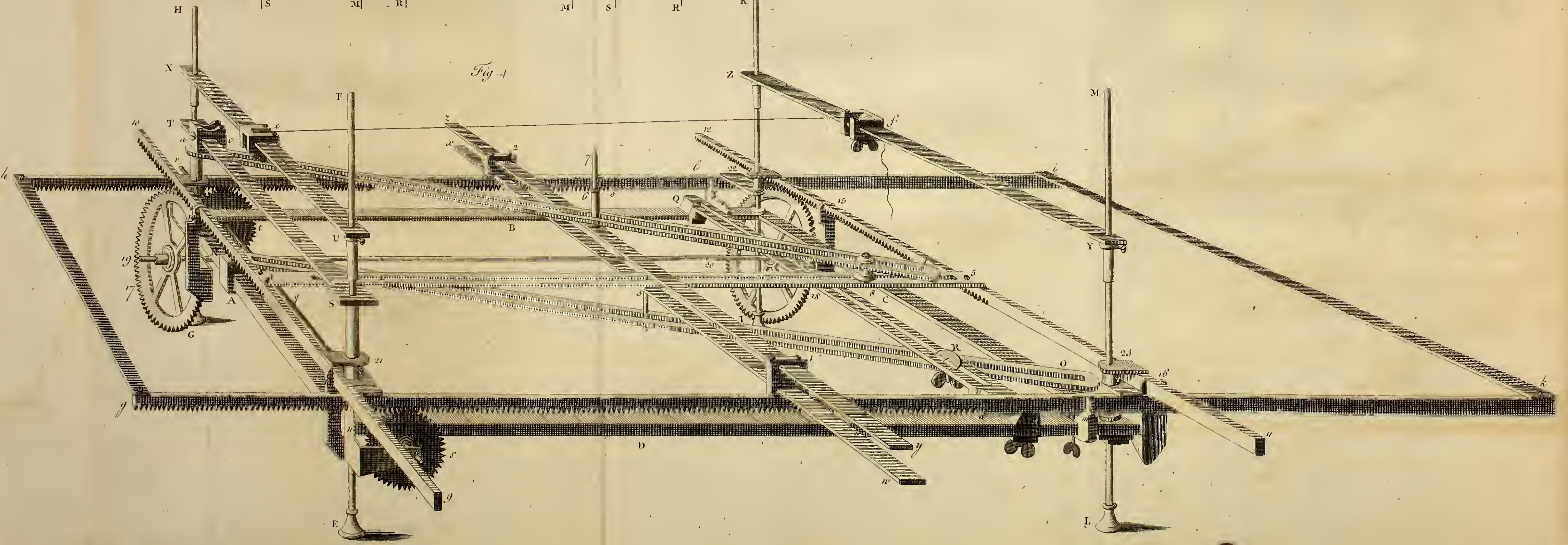


Fig. 4.



roots of the equation; and the ſeveral points, where the curve ſhall approach the baſe, but ſhall return without reaching it, will ſhew the *impoſſible* ones.

This is a method I myſelf fell into ten or twelve years ago, and have conſtantly uſed for finding the roots of ſuch equations as I have had occaſion to conſider. But his method is preferable to mine in one reſpect, *viz.* that whereas I always compute the value of the ordinates in numbers, he finds them by drawing certain right lines; however, when there are both poſſible and impoſſible roots in an equation, as generally there are, theſe methods are both of them extremely embarraſſing: the learned author therefore wiſhes, that ſome method might be thought of, whereby ſuch curves, as we are now ſpeaking of, might in all caſes be deſcribed by *local* motion; but this, he tells us, he looked upon as ſo very difficult a taſk, that he never attempted it. *Quod ad deſcriptionem attinet*, ſays he, *motum excogitare, quo tales accuratè deſignari poſſunt omnes [hujuſmodi curvæ] admodum difficile judico, quare id neque tentavi.* This hint, however, convinced me, that the thing was poſſible; I therefore determined to endeavour it.

I ſoon found, that if rulers were properly centered, and ſo combined together, that they ſhould always continue repreſentatives of the ſeveral right lines, by which he diſcovers the abovementioned ordinates, upon moving the firſt, a point or pencil, ſo fixed as to be carried along perpetually by the interſection of the firſt and laſt rulers, would deſcribe the required curve, let the number of dimenſions in the equation be what it will; only the greater that number, the greater muſt be the number of the rulers made uſe

 of.

of. And this appeared to me ſo obvious, that I wondered, neither the learned author, who ſeems to have the thing much at heart, nor any body elſe ſince the publication thereof, ſaw it.

Let the equation to be reſolved, be $a + bx + cxx + dxxx$, &c. $= o$.

Upon the line Z Z, as a baſe, in either the firſt or ſecond figure, draw the lines S S and R R perpendicular thereto, at any diſtance taken at pleaſure from each other. Then upon the line S S in either figure, Tab. VIII. ſet off the lines O A, A B, B C, C D, &c. proportionable to the coefficients a, b, c, d, &c. in the equation, remembering to take each line upwards from the extremity of the laſt, when the coefficient, it is to repreſent, is *affirmative*; but downwards from the ſaid extremity, when the coefficient is *negative*. Then through the extremity of the laſt of the lines O A, A B, B C, &c. which in our caſe is D, draw the line D c, parallel to the baſe Z Z, and through c, where D c interſects R R, draw c C; and parallel to S S, and at any diſtance from it taken at pleaſure, draw M M: then parallel to D c, and where C c interſects M M, draw the line $k b$; through b, where this laſt line interſects R R, draw b B; parallel alſo to D c, and where b B interſects M M, draw $l a$; and through a, where $l a$ interſects R R, draw a A. This done, let S S, R R and C c, be ſuppoſed to repreſent three rulers with grooves or notches cut quite through them, of ſuch form as is repreſented Fig. 3. and let thoſe rulers be ſcrewed down in their reſpective places S S, R R, and C c, to a plane or frame of ſufficient ſize.

Then

Then let other rulers of like form, as B*b*, A*a*, &c. be *moveable* upon the centers B, A, &c. and let thofe centers themfelves be moveable upwards and downwards in the groove or notch in the ruler SS, and in fuch manner, that the centers B and A may be placed upon one another, or upon C, if occafion requires, and let thofe centers be fcrewed down in their proper places, *viz.* the center A at A, the center B at B, &c. Then let *kb* and *la* reprefent other moveable rulers, fuch as the former, and fo confined that they fhall always continue parallel to themfelves and to the line D*c*; and let MM reprefent another ruler of like form. And let the rulers *kb* and MM be connected with the fixed ruler C*c* by a fliding pin paffing through the interfection of their notches at *q*, and let the rulers *kb*, B*b*, *la* and A*a* be connected with one another and with MM and RR, by like pins paffing through their notches at *b*, *r*, *a*, and *s*; and let the laft of thefe pins *s*, have the point of a pencil fixed therein; then, I fay, that if the ruler MM be moved backwards or forwards, from or towards SS, and always held parallel thereto (or be fo confined as to be capable of moving in no other manner) the pencil *s* fhall defcribe the required curve. And the diftances from the point O, at which that pencil fhall crofs the bafe ZZ, on the right hand fide of SS, fhall denote the *affirmative* roots of the equation; and the diftances from the fame point O, where it fhall crofs the faid bafe on the left hand fide of SS, fhall exprefs the *negative* ones; and the places, where it fhall approach the faid bafe, but fhall return without reaching it, fhall indicate the *impoffible* ones; the faid diftances being

always to be eſtimated on a ſcale on which the aſſumed line D c is *unity*.

Demonſt. Since the lines O A, A B, B C, &c. are to be taken *proportional* to the coefficients $a, b, c,$ &c. let us ſuppoſe the firſt of them, *viz.* O A, to be taken equal to the firſt coefficient a, or to any part of it taken at pleaſure; ſuppoſe for inſtance, to the n[th] part, that is, to $\frac{a}{n}$; then, to preſerve the above-mentioned proportionality, the next *viz.* A B will be equal to $\frac{b}{n}$, B C will be equal to $\frac{c}{n}$, and C D to $\frac{d}{n}$, &c. Call O Q or its equal D P, x; then Dc, as above-mentioned, being taken equal to *unity*, Pc will be equal to $1-x$, and D C being equal to $\frac{d}{n}$, and the triangles D C c and Pqc being ſimilar, we have this proportion, *viz.* $1 : 1-x :: \frac{d}{n} : \frac{d-dx}{n} = Pq$ or Dk; but kB is equal B C+C D−Dk, that is, to $\frac{c}{n}+\frac{d}{n}-\frac{d-dx}{n}$, that is, to $\frac{c+dx}{n}$; and by ſimilar triangles, as $kb : qb :: k\mathrm{B} : qr$, that is, in ſymbols, as $1 : 1-x :: \frac{c+dx}{n} : \frac{c+dx-cx-dxx}{n} = qr$ or kl: but Al is equal to A D−Dk−kl, that is, in ſymbols, to $\frac{b}{n}+\frac{c}{n}+\frac{d}{n}-\frac{d-dx}{n}-\frac{c+dx-cx-dxx}{n}$, that is, to $\frac{b+cx+dxx}{n}$; and by ſimilar triangles, $la : ra :: \mathrm{A}l : rs$; in ſymbols $1 : 1-x :: \frac{b+cx+dxx}{n} : \frac{b+cx+dxx-bx-cxx-dxxx}{n} = rs$; Q$s$ therefore, which by the figure is equal to

QP

QP − Pq − qr − rs, is equal to $\frac{a+b+c+d-d-dx}{n}$ $-\frac{c+dx-cx-dxx}{n}-\frac{b+cx+dxx-bx-cxx-dxxx}{n}$, that is, to $\frac{a+bx+cxx+dxxx}{n}$; confequently, when Qs is nothing, that is, when the curve defcribed by s, cuts the bafe, $\frac{a+bx+cxx+dx^3}{n}$ will be equal to nothing, and therefore equal alfo to $a+bx+cxx+dxxx$; this laft being alfo, from the equation propofed, equal to nothing; Qs therefore in thofe circumftances will be equal to $a+bx+cxx+dxxx$; and confequently whatever value of x or OQ renders $a+bx+cxx+dxxx$ equal to nothing, will render Qs equal to nothing: but every value of x that renders $a+bx+cxx+dxxx$ equal to nothing, is a root of the propofed equation $a+bx+cxx+dxxx=0$; confequently the curve will crofs the bafe ZZ at every *real* root of that equation, whether negative or affirmative, and therefore, as every one acquainted with curve lines knows, will attempt to do fo, but not quite reach it, at every *impoffible* one. Q, E. D.

This demonftration is adapted only to an equation of three dimenfions; but it is eafy to fee, it may be extended to any other.

Note. To obtain the negative roots, the rulers muft be extended to the left of the line SS, as reprefented fig. 2. where they are denoted by the fame letters as in the other figure; *viz.* the ruler Cc muft be extended from c to q; the ruler Bb from b to r, and aA from a to s, and onwards towards the left, the two laft turning upon the centers A, B, in the fixed line SS.

It

It is not neceſſary, that the curve ſhould be deſcribed with accuracy, or even that it ſhould fall upon the plane or frame, except where it croſſes or attempts to croſs the baſe; and therefore there will ariſe no inconveniency in this reſpect from taking the lines OA, AB, &c. large. But the immoveable rulers OD and Tc muſt be fixed ſo near together, that, their diſtance Dc or OT, being reckoned *unity*, the baſe OT extended to the extremity of the plane to the right, may take in all the *affirmative* roots: and when extended to the left, may take in all the *negative* ones.

There is one thing more, very well worth obſerving, *viz.* that if you have an equation of this ſort $xxx - 5xx + 1200x + 9000 = 0$ (where you may obſerve, that the coefficients unity, 5,1200 and 9000 are ſo very different from each other, that it would be difficult to ſet them off in due proportion upon the line OD) you may reduce them into a more manageable form, by the following contrivance: inſtead of each x in the equation put $10x$, $20x$ or $100x$, ſuppoſe $20x$; then inſtead of xxx you will have $8000xxx$, inſtead of $-5xx$ you will have $-2000xx$, &c. and the whole equation will ſtand thus, $8000xxx - 2000xxx + 24000x + 9000 = 0$; then divide each term by 100, and you bring it to $8xxx - 2xx + 24x + 9 = 0$, a much more tractable equation than the other: but then it muſt be remembered, that as you made x 20 times leſs than it was, the roots when diſcovered will be ſo many times leſs than the true ones, and therefore each muſt be multiplied by that number 20.

The

The following obſervations relative to the application of the abovementioned rulers may be uſeful:

1. The roots of equation are of three ſorts, *affirmative*, *negative*, and *impoſſible*; which laſt are ſometimes called *imaginary*.

2. Every equation contains as many roots as it has dimenſions.

3. The impoſſible roots go by pairs. Thus, if an equation has an impoſſible root of this form $a+b\sqrt{-1}$, it has another impoſſible one of this, *viz.* $a-b\sqrt{-1}$, which may be called its fellow: from hence it follows, that every equation, if it has any impoſſible roots, has either *two*, *four*, or *ſix*, &c. that is, an even number of them: and every time the curve deſcribed by the rulers abovementioned approaches the baſe, and leaves it again without croſſing it, it implies one impoſſible root and its fellow; ſo that if it approaches the baſe in this manner three times, it implies, that the equation has ſix impoſſible roots. And this is all the rulers can do with regard to this ſort of roots; they cannot ſhew *what* thoſe are, but only *how many* there are. I ſhall direct to a method of finding what they are in the 8th article below. Inaſmuch then, as the impoſſible roots go by pairs, and the number of roots in any equation is equal to the number of its dimenſions, it follows,

4. That every equation of an odd number of dimenſions, muſt contain at leaſt one *real* root.

5. Every equation whoſe firſt and laſt terms (when brought to one ſide) have contrary ſigns, will have at leaſt alſo one real root: and therefore, when this is the caſe, and the number of its dimenſions is

alſo *even*, it muſt have two real roots at leaſt: becauſe the number of dimenſions being *even*, and the number of impoſſible roots always *even*, the number of real ones muſt be *even* alſo.

6. If any equation be divided by the unknown quantity *minus* one of its roots, it will be reduced one dimenſion lower. And as every equation contains as many roots as it has dimenſions, it follows,

7. That if you deduct the number of impoſſible roots from the whole number of its roots, that is, from the number of its dimenſions, the remainder will give the number of its real roots.

8. When you have found by the rulers, what thoſe real roots are, put the unknown quantity (x) equal to each of them, tranſpoſe the terms in each equation to one ſide, multiply all the equations together, and divide the equation propoſed by their product; then make the quotient equal to nothing, and you have an equation containing all your impoſſible roots, without any real ones intermixed. Then thoſe impoſſible roots may be found by the method for that purpoſe laid down by Monſ. de *Bougainville* in his *Traité du Calcul integral*, in the fifth and ſixth chapters of his introduction; and which is the beſt method I know of.

His method conſiſts in parting the equation into two others, of the ſame number of dimenſions indeed, but ſuch as ſhall involve no other than real roots; which real roots you may then find by theſe rulers, or otherwiſe; and from thence you will obtain all the impoſſible roots of your equation. But becauſe few Engliſh mathematicians, I ſuſpect, are acquainted with

with this method, it may be uſeful to give the ſubſtance of it here in our own language.

The author previouſly demonſtrates the two following proportions.

Prop. I. That when a quantity is equal to nothing, and is compoſed of many terms, ſome of which are *real*, and the other are terms *multiplied by* $\sqrt{-1}$, the ſum of the real ones is equal to nothing; and the ſum of thoſe that are multiplied by $\sqrt{-1}$ is alſo equal to nothing. This is the 69th article of his introduction.

Prop. II. That when an equation involves imaginary roots only, the unknown quantity may always be ſuppoſed equal to $m+n\sqrt{-1}$; m and n being real quantities. This is the 80th article of his introduction.

Then to find the roots of ſuch an equation as we are ſpeaking of, for every unknown quantity (x ſuppoſe) in the equation, ſubſtitute $m+n\sqrt{-1}$, and you will obtain a new equation involving real terms, and terms multiplied by $\sqrt{-1}$; the former of which by Prop. I. are always equal to nothing, and ſo are the latter: make them ſo therefore, and you have two equations, from which the two aſſumed quantities m and n may be diſcovered; and conſequently, ſince the value of x is by the ſecond propoſition equal to $m+n\sqrt{-1}$, it is diſcovered alſo.

What I mean in the former part of this article may be explained by the following inſtance; ſuppoſe the real roots diſcovered by the rulers abovementioned to be a, b, $-c$, &c. then put $x=a$, $x=b$, $x=-c$, &c. tranſpoſe the terms to one ſide, and you have $x-a=0$, $x-b=0$, $x+c=0$, &c. multiply all theſe laſt

equations together, divide the equation propofed by their product, and proceed as abovementioned.

9. The greateft negative coefficient of any equation (confidered as affirmative) and increafed by unity always exceeds the greateft affirmative root of the equation. And therefore,

10. If for the unknown quantity (x) in the equation, you put that coefficient taken affirmatively and increafed by unity minus x, all the roots of the equation will be rendered affirmative. If you do this, you need only ufe fuch rulers, as are defcribed in the firft figure, whofe centers are at their extremities, and fo one fort of rulers will be fufficient for all cafes. For you may obferve, thofe in the fecond figure are different from the other, as to their centers.

11. If, when you have made all the roots of your equation affirmative, you would avoid removing the ruler M M to the right hand fide of R R, which might be attended with inconveniency; that is, if you would have all the roots of your equation fall between O and T, that is, between nothing and unity, inftead of the unknown quantity x in your laft equation, put x multiplied by the greateft negative coefficient therein confidered as affirmative and increafed by unity; for inftance, if the greateft negative coefficient in the equation be minus 9, put $10x$ inftead of every x in the equation, and you will have a new equation, all whofe roots fhall fall upon the line O T unproduced; for then they will be lefs than unity, that is, than D C or O T: but when the roots are thus found, each of them muft be multiplied by that coefficient increafed by unity, that is, in the above inftance, by 10; becaufe putting $10x$ for

for x, is making every root ten times ſmaller than it is.

Theſe propoſitions are all taken from the writings of the algebraiſts, or however are ſuch as eaſily follow from them, and therefore need not be demonſtrated here.

The following is a deſcription of a machine for regulating the motion of the abovementioned rulers, which I cauſed to be made by an excellent workman in this town, and which I deſire the ſociety to accept of, to be kept as a ſpecimen, for the inſpection of any gentleman, who may chuſe to have ſuch made. It extends only to equations of two dimenſions; but it is eaſy to ſee from it, how it may be carried to others of any number. A draft of it is exhibited in fig. 4. of the abovementioned table: where ABCD repreſents a frame of iron or ſteel, conſiſting of four ſtrait bars joined together at their extremities, and forming a rectangular parallelogram about 12 inches long and 8 broad; into which at its four corners are ſcrewed four perpendicular columns EF, GH, IK and LM, whoſe lower ends ſerve it as feet to ſtand on. And on one of the aforeſaid bars, *viz.* on A, is a moveable nut or ſlider, which may be ſcrewed to it at any point thereof; it appears in the figure at N; and on this nut, as a center, one end of the bar NO turns, whoſe other end is ſcrewed down to the croſs bar PQ at R, which croſs bar is ſcrewed down to the frame at P and Q, and may be ſet nearer or further from the end A at pleaſure: this bar repreſents the line RR in fig. 1. Then

 on

on the perpendicular pillars E F, G H, I K and L M, are fixed three bars S T, U X and Y Z; on the firſt of theſe S T, is a ſliding nut *c*, which carries one extremity of the bar *a b* turning upon it as a center; on the ſecond and third, *viz.* U X and Y Z, are alſo two nuts *e* and *f*, which may be ſcrewed on any part of them, and on which the ſilken cord *e f* is faſtned. The two firſt of thoſe bars, *viz.* S T and U X together with the bar A, or rather lines ſuch as the pricked line upon the upper one drawn upon them from pillar to pillar, repreſent the line SS in fig. 1. and the ſilken cord extended from the nut *e* to *f* and fixed to them, repreſents the baſe line Z Z in that figure.

Then beſides theſe there is another rectangular figure *g h i k*, about twice the length of the former, whoſe ſides, *g k* and *hi* ſlide in grooves or ſupporters ſcrewed to the frame ABCD, at proper places (three of which may be ſeen at *l*, *m*, and *n*), and have triangular teeth on their under ſides from *g* to *d*, and from *h* to *o*, which run in ſimilar teeth in two wheels *s* and *t*, of equal diameters, fixed on an axis *p q r*, and their axis *p r* is ſupported by proper cocks, one of which appears at *u*, the other not ſeen in this view of the figure. Theſe wheels being both fixed to the ſame axis, and the triangular teeth in the bars fitting thoſe in the wheels exactly, occaſion the bars *g k* and *h i* to move with equal paces, when the machine is put in motion; by which means the bars *wx* and *yz*, which ſlide in two pieces 1 and 2 ſcrewed to the ſaid bars *g k* and *hi*, neceſſarily move parallel to themſelves: theſe bars repreſent M M in fig.

fig. 1. In the lower of theſe bars $w x$ is fixed a perpendicular pin, at 3, whoſe upper end paſſes through the groove of the bar 4, 5, and its lower end through that of the bar N O, and in the upper of theſe bars $y z$, is fixed a perpendicular pin 6, 7, the upper end of which may be taken out, and a pencil put in its place; this pin repreſents the point s, and the former pin 3, the point r, in fig. 1. There is alſo a perpendicular pin ſcrewed to the bar 4, 5, at 8, which muſt be fixed upon that bar directly over the groove in the bar PQ. This pin denotes the point a in fig. 1. There are alſo two bars 9, 10, and 11, 12, with triangular teeth, which bars ſlide in ſupporters ſcrewed to the frame at proper places (which appear at 13, 14, 15, and 16), and ride upon the wheels 17 and 18, with ſimilar teeth, and both fixed on the axis 19, 20. Theſe wheels prevent the ſaid bars from moving with unequal paces, and therefore cauſe the bar 4, 5, which is ſcrewed down to them at each end of it, to move parallel to itſelf; this bar repreſents $l a$ in fig. 1. Then the aforeſaid nuts or ſliders e, f, c, N, and R, being ſcrewed down at proper places according to the coefficients of the equation (as ſhall be directed more particularly in the next article), and the bar $g h$ being puſhed forwards or backwards by the hand, will put the whole in motion; and the pin 6, 7, will deſcribe a curve, which ſhall be the *locus* of the equation, and the diſtances where it ſhall paſs under the ſilken cord $e f$, reckoning from the pricked line upon the bar U X, ſhall denote its real roots; and as many times as that pin ſhall approach the ſaid cord,

and

and then recede from it without paſſing under it, twice ſo many will be the number of impoſſible roots in that equation. N. B. There are ſome ſmall pieces ſcrewed on to the perpendicular pillars E F, G H, I K and L M, three of which appear at 21, 22, and 23; theſe ſerve only to prevent the bars which ſlide under them from riſing up.

To rectify the machine for a propoſed equation.

Screw the nuts *e* and *f*, which carry the ſilken cord at any equal diſtances from the pillars E F and L M. Then ſlide the nut *c*, which carries the extremity of the bar *a b*, till it is farther from the pillar E F than the place you have fixed the nut *e* at, by a number of diviſions upon any ſcale of equal parts, equal to the known term of the equation, if that term be affirmative; but ſo much nearer to it, if that term be negative; and fix it there. Then ſlide the nut N that carries the bar N O, till it is farther from or nearer to the pillar E F, than the laſt nut *c* is, by a number of diviſions taken from the ſame ſcale, equal to the ſecond coefficient of the equation (I mean that, where the unknown quantity is of one dimenſion); farther from it, if that coefficient is affirmative; but nearer, if it is negative: and fix it there. Then, laſtly, ſlide the nut R, which fixes the other end of the bar N O, till it is farther from a line drawn from the pillar E F to L M, that is, farther from the ſide D of the frame than the laſt nut N is from the ſame, as many diviſions, as the coefficient of that term of the equation, where the unknown quantity is of two dimenſions, indicates; farther,

ther, if that coefficient is affirmative, otherwife nearer to it: for which purpofe the end A of the frame, and the bars ST, UX, YZ and the crofs bar PQ fhould all be graduated, beginning at the front of the frame D. They are graduated on the machine itfelf in a manner fomewhat different, but it is found not fo convenient for ufe. Then the feveral diftances upon the cord *ef*, where the pencil or pin 6, 7, fhall crofs it, being reckoned from the pricked line upon the upper bar UX, and meafured upon a fcale, on which the diftance of the crofs bar PQ from a line drawn along the middle of the end A, from EF to GH, is *unity* (the reafon of which appears from the foregoing demonftration, where D*c* or OT in fig. 1. and which anfwers to the diftance of this line PQ from the bar A, was put equal to *unity*) fhall give the required roots. And if the cord *ef* is removed, and a piece of pafteboard put over the machine, on the two upper bars UX and YZ, having a ftrait line drawn on its under fide, reprefenting the cord *ef*, and a pencil with its point upwards be put in the place of the pin 7, that pencil will defcribe on the under fide of the pafteboard a curve, which with the faid right line fhall *conftruct* the equation propofed: and the larger the coefficients of the equation are (which coefficients may be made as large as you pleafe without altering the roots, by multiplying them by any number at pleafure) the larger will be the angles at which the curve and the ftrait line fhall cut each other; which in the conftruction of equations is a very defireable circumftance. And as it appears from the foregoing demon-

demonſtration, that this machine by the application of more bars, may be extended to equations of *all dimenſions*, it may not, I think, be improperly called an *Univerſal conſtructor of Equations*. I am

SIR,

With great reſpect,

your moſt humble and

moſt obedient ſervant,

London, Eſſex-ſtreet,
March 24, 1768.

J. Rowning.